Estimating Daily Surface Soil Moisture Using a Daily Diagnostic Soil Moisture Equation

Feifei Pan[1]

Abstract: One common problem is associated with water balance calculation methods for determining soil moisture for scheduling irrigation: errors in the estimated soil moisture are cumulative and frequent recalibrations are needed. A simple and robust approach to estimation of daily soil moisture using a daily diagnostic soil moisture equation is suggested and studied. The estimated soil moisture is a function of the time-weighted summation of the ratio of historical precipitation rate to soil moisture loss coefficient. To capture the seasonal variation in soil moisture loss coefficient, a sinusoidal wave function of the day of year (DOY) is used to represent the seasonal variation in loss coefficient. A 3-year continuous data set of daily soil moisture and daily precipitation collected at each of four Soil Climate Analysis Network sites—AR2091; in Arkansas, GA2013 in Georgia, NM2107 in New Mexico, and PR2052 in Puerto Rico—is applied to test the proposed method. The land cover/land use of these four sites is agricultural/crop fields, grasslands, or desert. Root mean square errors of the estimated volumetric soil moisture are less than 5% (v/v), and all correlation coefficients, R^2, are greater than 0.78. The results indicate that there are three advantages associated with the suggested approach: (1) errors in estimated soil moisture are noncumulative; (2) regular recalibration is not required; and (3) numerical iteration and initial moisture information are not required. **DOI: 10.1061/(ASCE)IR.1943-4774.0000450.**

CE Database subject headings: Soil water; Irrigation; Scheduling; Evapotranspiration; Drainage.

Author keywords: Soil moisture; Loss coefficient; Irrigation scheduling; Evapotranspiration; Drainage; Day of year; Diagnostic equation.

Introduction

In an arid or semiarid region, extremely scarce precipitation makes irrigation critical to agricultural production (Howell 2001). To improve water use efficiency (WUE) (Stanhill 1986), relatively accurate irrigation scheduling is very important (Jones 2004). Most irrigation scheduling methods can be classified into two types: soil moisture-based and plant-based (Jones 2004). Although plant growth directly depends on plant water status, and thus, a plant-based method could be more accurate than a soil moisture-based method (Jones 1990), the difficulties in automatically measuring or determining plant water stress make plant-based irrigation scheduling difficult and expensive to implement (Jones 2004). Therefore, soil moisture-based irrigation scheduling methods cannot be replaced by plant-based methods in the near future. Two types of approach are currently used to determine soil moisture levels for soil moisture-based irrigation scheduling (Jones 2004): direct soil moisture measurements (e.g., Campbell and Campbell 1982; Topp and Davis 1985) and soil water balance calculations (e.g., Allen et al. 1999). Direct soil moisture measurement is easy to use and relatively accurate, but expensive to implement because of soil heterogeneity requiring multiple soil moisture sensors to capture spatial variation in soil moisture (e.g., Pan and Peters-Lidard 2008). Soil water balance calculations are also easy to apply, but are not as accurate as direct measurement and require regular recalibration as errors in the estimated soil moisture are cumulative (Jones 2004).

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To overcome these challenges associated with the soil moisture methods commonly used for the purpose of irrigation scheduling, this research aims to develop a simple and robust approach for estimating daily soil on the basis of the method presented in Pan et al. (2003); the estimated soil moisture can be used for scheduling irrigation in the future. Pan et al. (2003) developed a simple method to retrieve surface soil moisture from rainfall observations based on a diagnostic equation of surface soil moisture derived from a linear stochastic partial differential equation (Entekhabi and Rodriguez-Iturbe 1994). The estimated soil moisture is a function of the time-weighted summation of ratio of historical rainfall rate to soil moisture loss coefficient (Pan et al. 2003). Using observations from three field campaigns in grassland/agricultural regions, that is, Monsoon ’90 (Schmugge et al. 1994), Washita ‘92 (Jackson and Le Vine 1996), and Southern Great Plains ‘97 (Jackson et al. 1999), Pan et al. (2003) were able to show that their simple method could retrieve surface soil moisture with a precision and accuracy comparable to those of remotely sensed soil moisture. However, in Pan et al. (2003), only soil moisture measurements collected during the summer seasons were tested. The seasonal variation in loss coefficient was neglected; therefore, the derived loss coefficient in Pan et al. (2003) cannot be used to estimate daily soil moisture accurately for all seasons. The objective of this study is to extend the work of Pan et al. (2003) and develop a soil moisture loss coefficient function that can be used for estimating daily soil moisture for all seasons.

Methods

Derivation of the Daily Diagnostic Equation of Surface Soil Moisture

On the basis of a linear stochastic differential equation suggested by Entekhabi and Rodriguez-Iturbe (1994), Pan et al. (2003) simplified the surface soil moisture dynamic equation to

$$z\frac{d\theta}{dt} = -\eta\theta + \gamma P \tag{1}$$

where z = thickness of the soil column; θ = soil moisture; $\eta\theta$ = loss of soil moisture; η = loss coefficient; P = precipitation rate; and γ = infiltration coefficient representing the ratio of infiltration to rainfall. Eq. (1) indicates that the surface soil moisture time change rate is equal to infiltration minus soil moisture loss. Vertical drainage and evaporation, or evapotranspiration (ET), are two main processes controlling surface soil water loss. Rearranging Eq. (1) results in

$$\frac{zd\theta}{-\eta\theta + \gamma P} = dt \tag{2}$$

Consider a time series of soil moisture at a point, as illustrated in Fig. 1, and integrate Eq. (2) between time t_2 and t_1 as

$$\int_{t_2}^{t_1} \frac{zd\theta}{-\eta\theta + \gamma P} = \int_{t_2}^{t_1} dt \tag{3}$$

For a shorter time step (≤ 1 day), the loss coefficient (η) and infiltration coefficient (γ) are assumed to be independent of time; that is, they are constants between time t_2 and t_1. P in Eq. (3) is the observed rainfall between time t_1 and t_2 and, thus, is independent of soil moisture. Under the earlier assumption, Eq. (3) becomes

$$-\frac{z}{\eta_1}\ln\left[\frac{\theta_1 - \gamma P_1/\eta_1}{\theta_2 - \gamma P_1/\eta_1}\right] = t_1 - t_2 \tag{4}$$

where η_1 and P_1 = loss coefficient and cumulative precipitation between time t_1 and t_2, respectively. Simplifying Eq. (4) yields

$$\theta_1 = \theta_2 e^{-\frac{\eta_1}{z}(t_1 - t_2)} + \frac{\gamma P_1}{\eta_1}\left[1 - e^{-\frac{\eta_1}{z}(t_1 - t_2)}\right] \tag{5}$$

For a daily time step (i.e., $t_1 - t_2 = 1$ day), Eq. (5) can be rewritten as

$$\theta_1 = \theta_2 e^{-\frac{\eta_1}{z}} + \frac{\gamma P_1}{\eta_1}(1 - e^{-\frac{\eta_1}{z}}) \tag{6a}$$

where η_1, P_1, and θ_1 = daily soil moisture loss coefficient, precipitation, and soil moisture on Day 1; and θ_2 = soil moisture of on Day 2. Day 2 is one day before Day 1. Similarly,

$$\theta_2 = \theta_3 e^{-\frac{\eta_2}{z}} + \frac{\gamma P_2}{\eta_2}(1 - e^{-\frac{\eta_2}{z}}) \tag{6b}$$

$$\theta_{n-1} = \theta_n e^{-\frac{\eta_{n-1}}{z}} + \frac{\gamma P_{n-1}}{\eta_{n-1}}(1 - e^{-\frac{\eta_{n-1}}{z}}) \tag{6c}$$

Substituting (6b), …, (6c) into Eq. (6a) results in

$$\theta_1 = \theta_n e^{-\sum_{i=1}^{i=n-1}(\eta_i/z)} + \sum_{i=2}^{i=n-1}\left[\frac{\gamma P_i}{\eta_i}(1 - e^{-\frac{\eta_i}{z}})e^{-\sum_{j=1}^{j=i-1}(\eta_j/z)}\right] + \frac{\gamma P_1}{\eta_1}(1 - e^{-\frac{\eta_1}{z}}) \tag{7}$$

Eq. (7) shows that as window size (i.e., n) increases, the exponential term $\exp\left[-\sum_{i=1}^{i=n-1}(\eta_i/z)\right]$ approaches a small number or zero; thus, the contribution of the leading term of the right-hand side of Eq. (7) to θ_1 diminishes. Therefore, at a threshold time window size n, soil moisture can be estimated directly from a weighted average of cumulative rainfall without any information on the initial soil moisture condition as

$$\theta_1 = \sum_{i=2}^{i=n-1}\left[\frac{\gamma P_i}{\eta_i}(1 - e^{-\frac{\eta_i}{z}})e^{-\sum_{j=1}^{j=i-1}(\eta_j/z)}\right] + \frac{\gamma P_1}{\eta_1}(1 - e^{-\frac{\eta_1}{z}}) = \gamma B \tag{8}$$

where B in Eq. (8) is defined as

$$B = \sum_{i=2}^{i=n-1}\left[\frac{P_i}{\eta_i}(1 - e^{-\frac{\eta_i}{z}})e^{-\sum_{j=1}^{j=i-1}(\eta_j/z)}\right] + \frac{P_1}{\eta_1}(1 - e^{-\frac{\eta_1}{z}}) \tag{9}$$

and represents the summation of the weighted ratio of rainfall rate to loss coefficient. Eq. (8) indicates that as the number of days before Day 1 increases, the contribution of the rainfall to the soil moisture of Day 1 is reduced because of the decreasing exponential term $\exp\left[-\sum_{j=1}^{j=i-1}(\eta_j/z)\right]$ in Eq. (8), which ensures that B approaches a stable value as n increases.

The threshold time window size depends on the value of (η/z) and the climate condition. Generally, volumetric soil moisture varies between 50 and 2%. If the annual soil water loss rate in the top 5-cm layer (i.e., $z = 5$ cm) is 1 m/year, it will take less than 3 months for the first term on the right-hand side of Eq. (7) to reach 0.5%. Therefore, a 3-month window is sufficient for calculating soil moisture using Eq. (8), without any initial condition of soil moisture in the climate region, where the annual potential evaporation or ET rate is greater than 1 m/year (e.g., in tropical and midlatitude regions). If the annual potential evaporation or ET rate is less than 1 m/year (e.g., in high-latitude areas), a larger window (i.e., $>$ 3 months) is needed.

Loss Coefficient

Similar to Entekhabi and Rodriguez-Iturbe (1994), the loss of soil moisture in Eq. (1) is approximated by the multiplication of soil moisture and the loss coefficient, $\eta\theta$. The loss (or dry-down) of surface soil moisture is controlled by drainage and evaporation (over bare ground) or ET (over vegetated land surface). Because drainage is controlled by soil hydraulic properties and ET is affected by the potential evapotranspiration (PET), the loss coefficient depends on both soil hydraulic properties (controlling drainage) and PET (controlling the actual ET rate).

Potential evapotranspiration is also known as atmospheric demand evapotranspiration, that is, evapotranspiration controlled by weather and climate conditions. For example, solar radiation is the energy that drives evaporation from bare soils and transpiration from vegetation. Air temperature and relative humidity directly affect the water vapor gradient between the atmosphere and the land surface. Wind speed controls the convection of water vapor from the land surface into the atmosphere. Canopy structure also affects

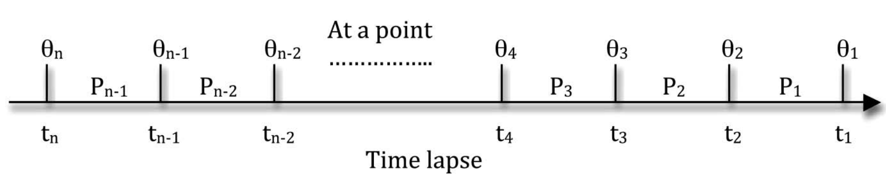

Fig. 1. Time series of soil moisture divided into $n-1$ periods; cumulative rainfall during each period is P_i, which is bounded by time t_i and t_{i+1}; θ_i is soil moisture at time t_i

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the vertical profiles of air temperature and wind, which, in turn, influence the exchange of water and energy between land surface and atmosphere. Many published methods estimate PET. Generally, these methods can be classified into three categories on the basis of data requirements (Jensen et al. 1990): (1) temperature-based methods, for which only air temperature and daylength are needed (e.g., Thornthwaite 1948; Hamon 1963); (2) radiation-based methods, for which net radiation and air temperature are needed (e.g., Priestley and Taylor 1972); and (3) combination methods, for which net radiation, air temperature, wind speed, and relative humidity are needed (e.g., Monteith 1965). In this study, a choice could be made among the preceding methods. However, incorporating weather conditions other than precipitation (e.g., solar radiation, air temperature, relative humidity, and wind speed) would make the approach complicated and difficult to implement, as such weather condition data may be not available in all geographic locations. Because climate conditions (e.g., daily mean values of solar radiation, air temperature, and relative humidity at a location) are approximate functions of the day of year (DOY), a sinusoidal wave function of DOY is used to represent the daily soil moisture loss coefficient η (which depends on soil hydraulic properties and PET rate) as

$$\eta_i = c_1 + c_2 \sin\left[\frac{2\pi(\mathrm{DOY}_i + c_3)}{365}\right] \tag{10}$$

where η_i = loss coefficient of day i; DOY_i = DOY of day i; and c_1, c_2, and c_3 = constants, hereafter referred as the loss coefficient parameters. These three loss coefficient parameters, depending on geographic location, soil, and vegetation characteristics, can be inversely determined by maximizing the coefficient of correlation between observed soil moisture and B value (i.e., best fit between observed soil moisture and B value), given as

$$\max\left\{\frac{\sum_{i=1}^{m}[(\theta_i - \bar{\theta})(B_i - \bar{B})]}{\sqrt{\sum_{i=1}^{m}(\theta_i - \bar{\theta})^2}\sqrt{\sum_{i=1}^{m}(B_i - \bar{B})^2}}\right\}\Bigg|_{c_1,c_2,c_3} \tag{11}$$

where θ_i and B_i, $i = 1, \ldots, m$ = soil moisture measurements and computed B values, respectively; and $\bar{\theta}$ and $\bar{B}$ = mean values of soil moisture measurements and computed B values, respectively.

Two methods are often used to achieve the best fit: the Gauss–Newton method (Fletcher 1987) and simple "global search" methods. To use the Gauss–Newton method to maximize the correlation coefficient given in Eq. (11), a Jacobian matrix must first be constructed. However, the three unknown parameters (c_1, c_2, and c_3) in the loss coefficient function [Eq. (10)] and the high nonlinearity of the B expression [Eq. (9)] make it difficult to obtain an analytical expression of the Jacobian matrix. Compared with the Gauss–Newton method, global search methods are simple.

Several methods can be used for a global search. The simplest method is to exhaustively search the parameter space of unknown parameters. Although this method is simple and efficient for problems with few parameters, the computation becomes unfeasible for problems with many parameters (Mosegaard and Tarantola 1995). To inverse a nonlinear equation with high dimensionality (i.e., many parameters), a Monte Carlo search, that is, a random walk in the parameter space, can be used. Although the concept underlying the Monte Carlo method is simple and not new, it has been widely used to solve inverse problems, especially in seismology (Keilis-Borok and Yanovskaya 1967; Rothman 1986; Landa et al. 1989; Mosegaard and Vestergaard 1991; Koren et al. 1991). Because the primary objective of this paper is to demonstrate the feasibility of the suggested method, the simple and feasible Monte Carlo search method is used here.

According to Eq. (10), c_1 represents the mean value of the loss coefficient, c_2 is the magnitude of the loss coefficient variation, and c_3 is the phase of the sinusoidal wave. Because the loss coefficient cannot be negative (i.e., always greater or equal to zero), c_2 must be less than or equal to c_1 Both c_1 and c_2 are in the same units as precipitation (i.e., length/day, because a daily time step is used in this study), and c_3 is expressed in DOY. On the basis of the map of mean annual pan evaporation for the contiguous United States of Farnsworth and Thompson (1982), the maximum free-water-surface evaporation is approximately 0.97 cm/day (140 in./year) in southeast Arizona. Therefore, a maximum soil moisture loss coefficient of 2 cm/day is sufficiently large to include all climate conditions in the tropical and middle-latitude regions. Thus the searching domain of the loss coefficient function parameters is given as

$$\text{searching domain} = \{0 < c_1 < 2\ \text{cm/day};\ \ 0 < c_2 \le c_1;\ \ 0 < c_3 < 366\} \tag{12}$$

Relationship between Soil Moisture and B Value

To use Eq. (8) to estimate soil moisture over the dynamic range of soil moisture (i.e., between residual soil moisture and saturated soil moisture), the infiltration coefficient γ must be determined. However, in reality, the infiltration coefficient γ cannot be considered a single constant as it may vary with soil moisture. As B increases, soil moisture increases and approaches saturated soil moisture, and the infiltration coefficient γ will decrease and finally approach zero (Pan et al. 2003). The decrease in infiltration coefficient with increase in B determines that an exponential curve is the best fit of soil moisture versus B (Pan et al. 2003). Therefore, the general form of soil moisture as a function of B should be

$$\theta = \theta_{re} + (\phi_e - \theta_{re})(1 - e^{-c_4 B}) \tag{13}$$

where θ_{re} and ϕ_e = effective residual soil moisture and effective porosity, respectively; and c_4 = empirical constant related to soil hydraulic properties. The infiltration coefficient γ loses its role in determining soil moisture. Eq. (13) is called the daily diagnostic soil moisture equation.

Study Sites and Data

The Soil Climate Analysis Network (SCAN), a comprehensive, nationwide soil moisture and climate information system, is administrated by the U.S. Department of Agriculture Natural Resources Conservation Service (USDA NRCS) through the National Water and Climate Center (NWCC), in cooperation with the NRCS National Soil Survey Center (NSSC) (Seyfried et al. 2005; Schaefer et al. 2007). The SCAN system measures soil moisture content hourly at 5, 10, 20, and 50 cm and atmospheric forcing (e.g., precipitation, air temperature, solar radiation). The archived data at each SCAN site can be downloaded from http://www.wcc.nrcs.usda.gov/scan.

Because the primary objective of this study is to develop an approach to estimate soil moisture for future irrigation scheduling purpose, four sites (see Table 1) in agricultural fields or grasslands were chosen from more than 100 SCAN sites to demonstrate the approach and methodology described under "Methods." On the other hand, to simplify the problem, snow processes (i.e., snow accumulation and snow melting) are not considered in this study. Therefore, four sites were chosen from those where snow accumulation during the study period is zero.

Table 1. Four SCAN Sites

Site	State/region	Lat.	Long.	Land cover	Soil texture	Par. est. period	Testing period
AR2091	Arkansas	34°17′N	91°21′W	Grass	Silt loam	1/1/07–12/31/08	1/1/09–12/31/09
GA2013	Georgia	33°53′N	83°26′W	Grass/crop	Sandy loam	1/1/06–12/31/07	1/1/08–12/31/08
NM2107	New Mexico	33°32′N	103°15′W	Desert	Loamy sand	1/1/07–12/31/08	1/1/09–12/31/09
PR2052	Puerto Rico	18°28′N	67°3′W	Grass/bare soil	Clay	1/1/07–12/31/08	1/1/09–12/31/09

Results and Discussion

At each site, a 3-year record of continuous daily rainfall and top-5-cm soil moisture was compiled. Daily soil moisture and daily precipitation data from the first 2 years were used for parameter estimation, and data from the third year were used to test the suggested method. As described under "Methods," to apply the diagnostic soil moisture equation to estimation of soil moisture, the loss coefficient must first be determined as a sinusoidal wave function of DOY [i.e., Eq. (10)]. The three loss coefficient parameters were determined by maximizing the coefficient of correlation between observed soil moisture in the parameter estimation period and computed B values using the Monte Carlo search method (Mosegaard and Tarantola 1995) as described under "Methods." Table 2 lists the results and the associated coefficients of correlation ($R^2_{\theta,B}$) between observed soil moisture and B values. Scatterplots of observed soil moisture versus B are shown in Fig. 2. Both the scatterplots in Fig. 2 and the associated high $R^2_{\theta,B}$ values (all $R^2_{\theta,B} \geq 0.7$) indicate that: (1) the proposed sinusoidal wave function of the loss coefficient [Eq. (10)] can capture the seasonal variation in the soil moisture dry-down process; and (2) the relationship between B value and soil moisture, that is, the proposed diagnostic soil moisture [Eq. (13)], can be used to estimate soil moisture without any information on initial soil moisture condition (Pan et al. 2003).

Table 2. Estimated Loss Coefficient Parameters and Coefficients of Correlation ($R^2_{\theta,B}$) between Observed Soil Moisture and Computed B Values

Site	c_1	c_2	c_3	$R^2_{\theta,B}$
AR2091	0.231	0.177	268	0.70
GA2013	0.460	0.178	274	0.74
NM2107	0.553	0.378	261	0.79
PR2052	0.530	0.181	285	0.78

In the diagnostic soil moisture equation [Eq. (13)], three parameters—effective residual soil moisture, θ_{re}; effective porosity, ϕ_e; and parameter c_4—can be determined by best-fitting the scatterplot of observed soil moisture versus B value in Fig. 2 using the least-squares method. The Matlab Curve Fitting Toolbox was used to perform the best-fitting, and the best-fit curves obtained are plotted in Fig. 2. Estimated effective residual soil moisture, effective porosity, parameter c_4, root mean square errors (RMSEs) and coefficients of correlation between the observed and estimated soil moisture ($R^2_{\theta,\theta'}$) are listed in Table 3. The time series plots of the observed and estimated soil moisture in the parameter estimation period are shown in Fig. 3. The results indicate that there is a good agreement between observed and computed soil moisture in the parameter estimation period because of the small errors (all RMSEs are $< 5\%$) are high correlation coefficients (all $R^2_{\theta,\theta'}$ values are ≥ 0.8).

To carry out an additional test of the suggested method, the derived loss coefficient function [Eq. (10)] and the effective hydraulic properties and parameters in the diagnostic soil moisture equation [Eq. (13)] were used to estimate soil moisture in the method-testing period (i.e., the third year) at each site. Time series plots of observed and estimated soil moisture in the testing period are shown in Fig. 4. The RMSEs and correlation coefficients of estimated soil moisture during the method-testing period at each site are listed in Table 4. Agreement between observed and estimated soil moisture in the third year (the method-testing period) is also good; that is, all

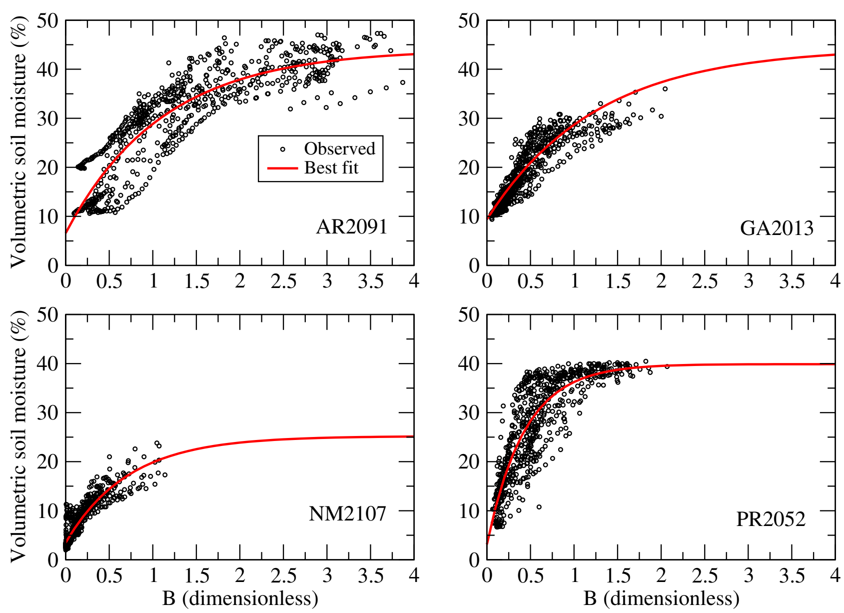

Fig. 2. Scatterplots of observed top-5-cm soil moisture versus B values, and best-fit curves of the scatterplots at AR2091, GA2013, NM2107, and PR2052

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Table 3. Estimated Effective Soil Hydraulic Properties, RMSEs, and Coefficients of Correlation between Observed and Estimated Soil Moisture during the Parameter Estimation Period

Site	θ_{re}	ϕ_e	c_4	RMSE	$R^2_{\theta,\theta'}$
AR2091	6.6	44.1	0.9	4.50	0.80
GA2013	9.4	44.4	0.8	2.73	0.82
NM2107	3.5	25.2	1.4	1.72	0.85
PR2052	3.1	39.9	1.4	4.91	0.86

Table 4. RMSEs and Coefficients of Correlation between Observed and Estimated Soil Moisture during Method-Testing Period

Site	RMSE	$R^2_{\theta,\theta'}$
AR2091	3.75	0.78
GA2013	3.07	0.79
NM2107	1.66	0.80
PR2052	4.03	0.78

RMSEs are less than 5%, and the correlation coefficient $R^2_{\theta,\theta'}$ is between 0.78 and 0.80 (Table 4).

As Jones (2004) indicated, there is a common problem associated with water balance calculation methods: the errors in the estimated soil moisture are cumulative and regular recalibration is needed. To demonstrate that the method suggested in this paper can overcome this problem, the root mean square error of each month ($RMSE_m$) is calculated as

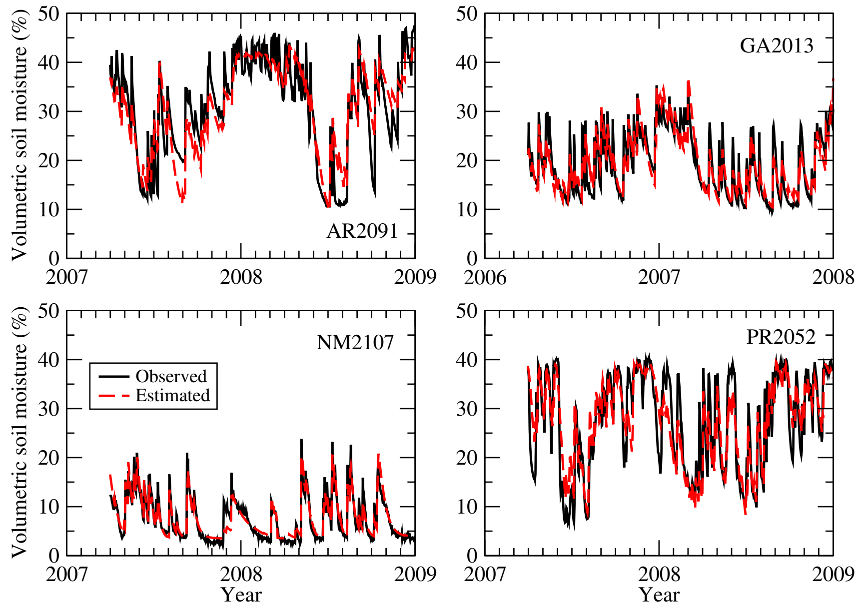

Fig. 3. Observed and estimated top-5-cm soil moisture during the parameter estimation period; because a 3-month window is used, estimated soil moisture starts on April 1 at each site

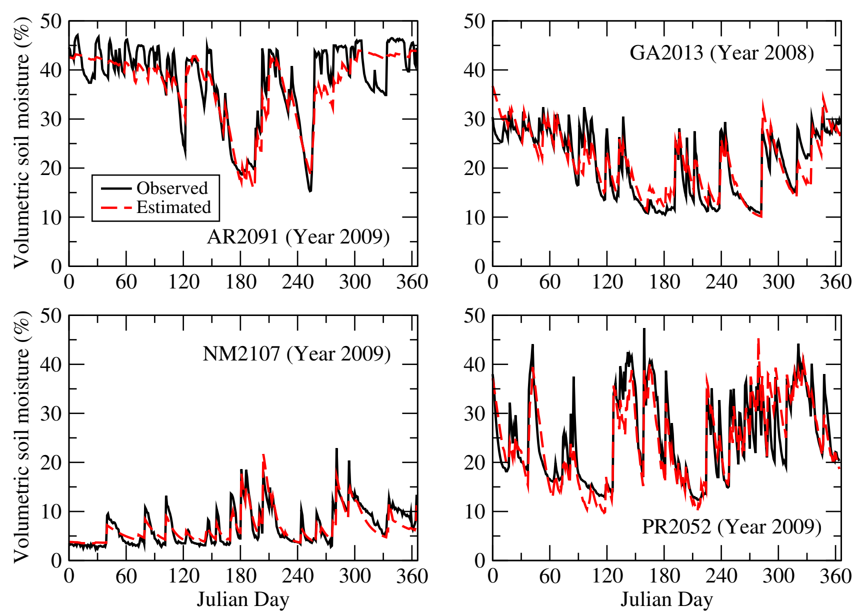

Fig. 4. Observed and estimated top-5-cm soil moisture during the method-testing period

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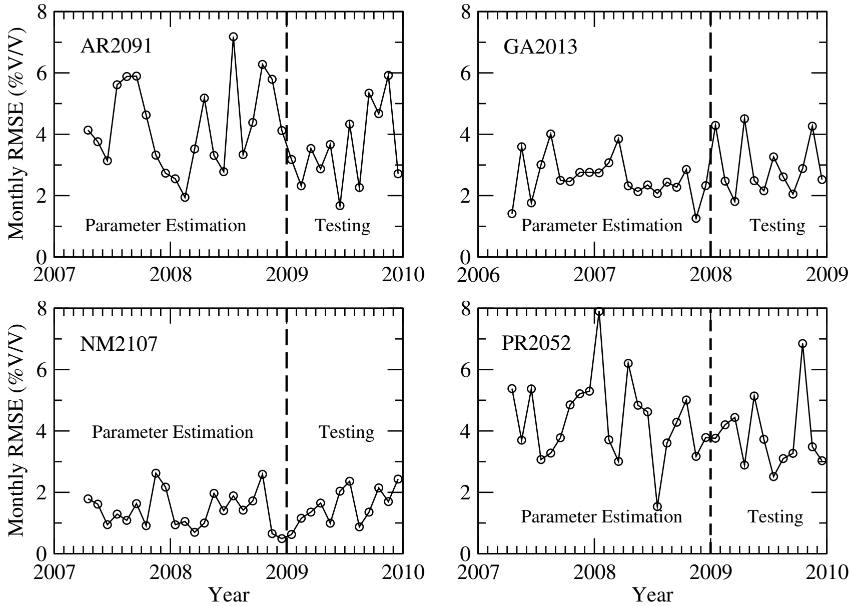

Fig. 5. Root mean square errors of estimated soil moisture over parameter estimation period (left-hand side of vertical dashed line) and method-testing period (right-hand side of vertical dashed line)

$$\mathrm{RMSE}_m = \sqrt{\frac{\sum_{i=1}^{d_m}(\theta_i' - \theta_i)^2}{d_m}} \tag{14}$$

where d_m = total number of days in month m; θ_i = observed soil moisture; and θ_i' = estimated soil moisture of day i in month m. The time series plots of RMSE_m in Fig. 5 indicate that the errors in estimated soil moisture are not cumulative; therefore, no recalibration is needed.

Conclusions

A simple and robust approach to estimation of daily soil moisture using a daily diagnostic soil moisture equation has been suggested and tested. The daily diagnostic soil moisture equation is derived from the linear stochastic partial differential equation of soil moisture dynamics (Entekhabi and Rodriguez-Iturbe 1994; Pan et al. 2003). The estimated soil moisture is a function of the time-weighted summation of ratio of historical rainfall rate to soil moisture loss coefficient. To capture the seasonal variation in soil moisture loss coefficient, a sinusoidal wave function of the day of year is used to represent the loss coefficient. The optimal parameters of the sinusoidal wave function are inversely determined by maximizing the coefficient of correlation between the observed soil moisture and the time-weighted summation of ratio of historical rainfall rate to soil moisture loss coefficient. The observed soil moisture data collected at the four USDA SCAN sites were used to test the proposed method. The small errors (RMSEs $< 5\%$) and high correlation coefficients (> 0.8) of the estimated soil moisture indicate three advantages associated with the proposed approach: (1) the errors in estimated soil moisture are noncumulative; (2) regular recalibration is not required to correct for cumulative errors; and (3) numerical iteration and initial moisture inputs are not needed. Therefore, the daily surface soil moisture diagnostic equation approach is more efficient than the traditional numerical modeling approach.

Because this study focused on the feasibility of the suggested method, the simple sinusoidal wave function [Eq. (10)], with only one independent variable (DOY) and three loss coefficient parameters (c_1, c_2, and c_3), was used to approximate the soil moisture loss coefficient. The three loss coefficient parameters depend on geographic location, soil, and vegetation characteristics. Because only four SCAN sites were chosen for testing the approach in this study, no effort was made to establish the relationship among loss coefficient function parameters, geographic location, soil properties, and vegetation characteristics and to determine the dependency of effective residual soil moisture, effective porosity, and parameter c_4 in the daily diagnostic soil moisture equation on soil and topographic characteristics, which deserve a future study.

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