

# Generating and updating multiplicatively weighted Voronoi diagrams for point, line and polygon features in GIS

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## Abstract

A Voronoi diagram is an interdisciplinary concept that has been applied to many fields. In geographic information systems (GIS), existing capabilities for generating Voronoi diagrams normally focus on ordinary (not weighted) point (not linear or area) features. For better integration of Voronoi diagram models and GIS, a raster-based approach is developed, and implemented seamlessly as an ArcGIS extension using ArcObjects. In this paper, the methodology and implementation of the extension are described, and examples are provided for ordinary or weighted point, line, and polygon features. Advantages and limitations of the extensions are also discussed. The extension has the following features: (1) it works for point, line, and polygon vector features; (2) it can generate both ordinary and multiplicatively weighted Voronoi diagrams in vector format; (3) it can assign non-spatial attributes of input features to Voronoi cells through spatial joining; and (4) it can produce an ordinary or a weighted Euclidean distance raster dataset for spatial modeling applications. The results can be conveniently combined with other GIS datasets to support both vector-based spatial analysis and raster-based spatial modeling.

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## 1. Introduction

Given a set of a finite number of distinct points in the 2-D Euclidean space, a Voronoi diagram of the point set is a collection of regions that divide up the plane, and all locations in one region (except on the region boundary) are closer to the corresponding point than to any other point (Fig. 1). Voronoi diagrams are named after the Russian mathematician Georgy Fedoseevich Voronoi who defined and studied the general  $n$ -dimensional case

in 1908. Discussions on the concept of Voronoi diagram from both historical and geometric viewpoints can be found in Okabe et al. (1992, 2000).

Voronoi diagrams have been widely applied to space partition problems in various disciplines, from astronomy to geography to zoology. Okabe et al. (2000) listed 22 fields in which applications of Voronoi diagrams can be found, and it is believed that the application fields are not limited to the list. Recent application examples of Voronoi diagrams include Voronoi-based cellular automata (Shi and Pang, 2000), 3-D adaptive tomography in geophysical prospecting (Böhm et al., 2000), earthquake distribution analysis and early warning

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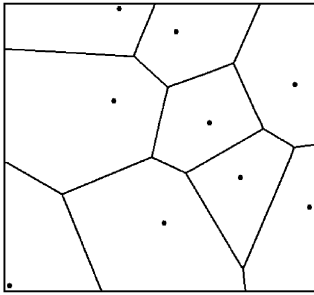


Fig. 1. A Voronoi diagram of points.

(Nicholson et al., 2000; Cua, 2004; Satriano et al., 2006), exposure analysis of wireless sensor network (Meguerdichian et al., 2001), reservoir modeling in petroleum exploitation (Subbeya et al., 2004), environmental risk assessment (Greco et al., 2005), remote sensing mapping accuracy assessment (Nelson et al., 2005), biological structure characterization (Costa et al., 2006), and domain boundary reconstruction for point datasets (Kolingerová and Zalik, 2006). In geophysics, Iervolino et al. (2006) discussed real-time risk analysis for earthquake early warning where magnitude estimation is performed via the Bayesian approach, and the earthquake localization is based on the Voronoi cells. In geography, some of the early applications of Voronoi diagrams in market area analysis can be dated back to 1841 (Shieh, 1985; Okabe et al., 2000). Later examples include Boots (1980, 1986) and Boots and Getis (1988), among many others. Economic geography and urban geography problems are major application areas of Voronoi diagrams in geography, for example, modeling retail trade areas (Boots and South, 1997) and solving locational optimization problems (Okabe et al., 1992, 2000; Okabe and Suzuki, 1997). Since the 1990s, Voronoi diagram has received increased attention as a spatial data model in GIS (Gold, 1991, 1994, 2004; Okabe et al., 1994; Gold and Condal, 1995; Anton et al., 1998; Li et al., 1999; Chakroun et al., 2000; Chen et al., 2001; Boots et al., 2003; Gold and Angel, 2006; Ledoux and Gold, 2006; Masuyama, 2006).

Although theoretical and computational aspects of Voronoi diagrams have been extensively documented, limited efforts have been made to bring Voronoi diagrams into GIS for practical use. There are 2 major issues concerning the integration of Voronoi diagrams and GIS: (1) many approaches to

generating Voronoi diagrams have worked with point data outside GIS, and it is not straightforward to bring the results into GIS (for example, Reitsma et al., 2004; Ohyama<sup>1</sup>); (2) in many applications, the location of a feature alone is not enough, and the importance (or weight) of the feature should be taken into account. Many existing methods in GIS have focused on ordinary point features, but not weighted features including points, line segments, and polygons. Examples of these issues can be found in literature and from GIS software products. Lomas<sup>2</sup> created a Thiessen Polygon Dynamic Link Library (DLL) for the ArcGIS software developed by the Environmental Systems Research Institute (ESRI), to generate Voronoi polygons. But the DLL only works for point data, and weights are not taken into account. In ArcGIS 9.1 released by ESRI in 2005, ordinary Voronoi diagrams (OVDs) can be generated in raster format using the Euclidean Allocation tool in the Spatial Analyst extension, or displayed in vector format using the Geostatistical Analyst extension, or created from a “store layer” (point dataset) in the Business Analyst extension. However, none of the extensions can create weighted Voronoi diagrams. Several references are noteworthy in bringing weighted Voronoi diagrams into GIS. Tiefelsdorf and Boots<sup>3</sup> developed a program GAMBINI which calculates, draws and exports multiplicatively weighted Voronoi diagrams on Windows 3.1 and Windows 95/NT. GAMBINI is loosely linked to GIS via input and output files, and Voronoi polygons need to be manually created from the imported arcs, which can be a time-consuming process. Using vector data structure and algorithms, Gahegan and Lee (2000) studied 6 specific Voronoi diagram variants, including multiplicatively weighted Voronoi diagrams, to support interactive spatial analysis. They concluded that, compared with OVDs, the weighting of generator points allows for modeling a wide range of situations, but the reconstruction algorithm is more complex. Mu (2004) demonstrated the composition and decomposition of multiplicatively weighted Voronoi polygons, and proposed using decomposition to

<sup>1</sup>Takashi Ohyama's webpage: <http://www.nirarebakun.com/voro/emwvoro.html>.

<sup>2</sup>Lomas, T., Create Thiessen Polygons 3.0: <http://arcscript.esri.com>.

<sup>3</sup>Tiefelsdorf, M., Boots, B., 1997. GAMBINI Multiplicatively Weighted Voronoi Diagram. <http://info.wlu.ca/%7Ewwwgeog/facstaff/boots.htm>.

understand landscape processes. It seems that all these references on weighted Voronoi diagrams in GIS are for point generators, and the methods for generating Voronoi diagrams are all vector-based. In fact, as reviewed by Li et al. (1999), vector-based methods for generating Voronoi diagrams work well for point datasets, but are not efficient for line segments and area features. In addition to point features, it is desirable to generate Voronoi diagrams for line segments and area features to support certain applications. Okabe and Suzuki (1997) noticed that the locational optimization problems of line-like and area-like facilities are less developed and worth challenging. In geosciences, weighted Voronoi diagrams for line segments and area features can be very useful in some applications. For example, the zone of influence for active faults may be assessed based on the length of the fault segments, and target areas in mineral exploration can be delineated depending on the size of alteration zones.

For better integration of Voronoi diagrams and GIS to support various applications, it is important to bridge the gap between the above-mentioned issues. The objective of this research is to develop a raster-based approach to generating and updating both ordinary and multiplicatively weighted Voronoi diagrams for point, line, and polygon features in a GIS environment. Implemented as an ArcGIS extension, the approach accepts point, line, and polygon features in shapefile vector format, calculates ordinary or weighted Euclidean distances in a raster environment, and finally creates an output Voronoi diagram as a polygon shapefile and an ordinary or weighted distance raster. Optionally, non-spatial attributes of input features can be assigned to output Voronoi diagrams through spatial joining. Therefore, the output from the extension can be used for both vector-based spatial analysis and raster-based spatial modeling. The major reason for employing the raster-based approach is that point, line, and polygon features can be treated in the same manner when calculating distance rasters using ArcObjects, which greatly simplifies the process for generating Voronoi diagrams. Following this introduction, Section 2 describes the methodology and implementation of the extension, Section 3 provides examples for generating and updating ordinary and weighted Voronoi diagrams, followed by discussions in Section 4, and conclusion in Section 5.

## 2. Methodology and implementation

### 2.1. Generating multiplicatively weighted Voronoi diagrams

In Fig. 1, it is assumed that each generator point has the same weight. In many spatial analysis and modeling problems, generators have different weights due to the differences in their properties. For example, earthquakes have different magnitudes, and cities have different population sizes. Okabe et al. (2000) discussed 4 types of weighted Voronoi diagrams: multiplicatively weighted, additively weighted, compoundly weighted, and power diagram. This study focuses on multiplicatively weighted Voronoi diagrams.

Given 2 generators A and B with weights  $w_a$  and  $w_b$  ( $w_a > 0$ ,  $w_b > 0$ ), respectively, we can describe the Euclidean distances to A and B using 2 upside-down right circular cones (Fig. 2), as if 2 trees A and B are competing with each other for canopy space. In order to show the Euclidean distance from every point location to a generator, we use a raster data model to represent distance values. Therefore, each cone represents the distance raster surface for the generator. The 2 cones touch each other at Q.

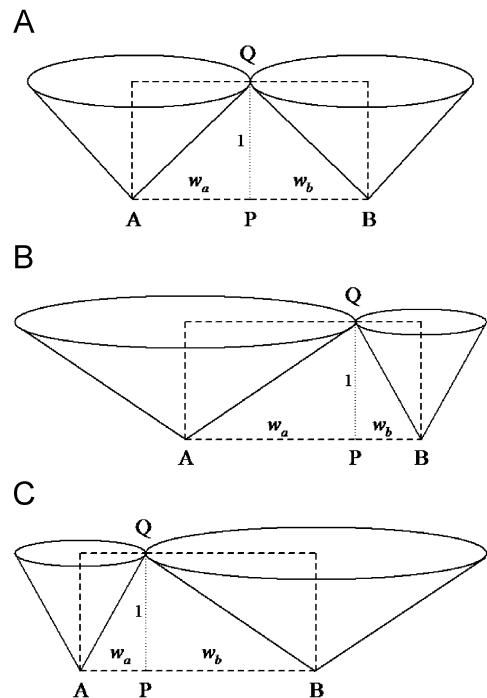


Fig. 2. Distance surfaces for (A)  $w_a = w_b$ ; (B)  $w_a > w_b$ ; and (C)  $w_a < w_b$ .

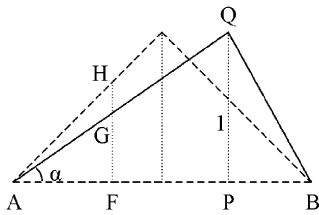


Fig. 3. Relationship between ordinary Euclidean distance (OED) and weighted Euclidean distance (WED).

For simplicity, let the raster cell value at point Q be equal to unit length 1 ( $\overline{PQ} = 1$ ). If  $w_a = w_b$ , the 2 rasters represent “ordinary Euclidean distance (OED)” (Fig. 2A). Otherwise, they represent “weighted Euclidean distance (WED)” (Fig. 2B, C).

What is the relationship between OED and WED for the generator point A? This can be answered by analyzing Fig. 3, a composite of the triangle ABQ in Fig. 2A and the triangle ABQ in Fig. 2B. For a random point F between A and B (Fig. 3),  $\overline{FH}$  is OED,  $\overline{FG}$  is WED, and  $\overline{FH} = \overline{AF}$ . Since A and B are multiplicatively weighted generators,  $\overline{AP}$  should be proportionally related to  $w_a$ , that is,  $\overline{AP} = cw_a$  ( $c > 0$ ). Obviously, we have

$$\tan \alpha = \frac{\overline{PQ}}{\overline{AP}} = \frac{\overline{FG}}{\overline{AF}} \tag{1}$$

From (1) we have:

$$\overline{FG} = \frac{\overline{PQ} \cdot \overline{AF}}{\overline{AP}} = \frac{\overline{FH}}{cw_a} \tag{2}$$

Because  $c$  is a constant, Eq. (2) can be simplified to

$$\overline{FG} = \frac{\overline{FH}}{w_a} \tag{3}$$

Eq. (3) shows that WED for generator A can be calculated using OED divided by the weight  $w_a$ . Denote  $g(x, y)$  as the WED raster, and  $f(x, y)$  as the OED raster, Eq. (3) can be expressed as

$$g(x, y) = \frac{f(x, y)}{w_a} (w_a > 0). \tag{4}$$

Similar expressions can be obtained for generator B. Fig. 4 illustrates a WED raster surface profile for 3 generators A, B, and C with weights  $w_a$ ,  $w_b$ , and  $w_c$ , respectively, and  $w_b > w_a > w_c$ . The shaded portions in Fig. 4 represent the final WED raster surface profile after all generators are taken into account. From Fig. 4 it can be seen that the final WED raster,  $R(x, y)$ , can be obtained by calculating

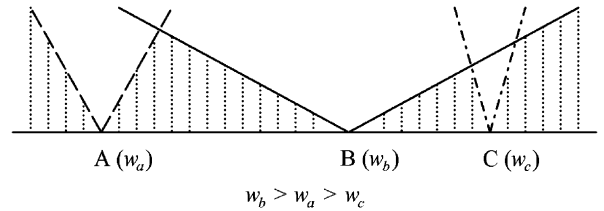


Fig. 4. Weighted Euclidean distance (WED) raster surfaces for 3 generators.

cell-by-cell minimum of the WED rasters obtained from all generators:

$$R(x, y) = \min[g_i(x, y)] (i = 0, 1, 2, \dots, k - 1), \tag{5}$$

where  $x$  and  $y$  are individual cell locations, and  $k$  is the total number of generators. It should be noted that the min function in Eq. (5) works on the cell-by-cell basis where  $x = 0, 1, 2, \dots, m - 1$  and  $y = 0, 1, 2, \dots, n - 1$  ( $m$  is the number of columns and  $n$  is the number of rows in a raster). In actual implementation, it may not be feasible to store  $k$  rasters in memory in order to calculate  $R(x, y)$ , especially if a large number of features are involved for generating Voronoi diagrams. This problem can be solved by comparing the distance rasters in a loop structure throughout all features (generators), as described in the following pseudo-code:

- $k$  – The number of features.
- $i$  – Feature index.
- $r_0(x, y)$  – The 1st raster for ordinary Euclidean distance.
- $s_0(x, y)$  – The 1st raster for weighted Euclidean distance.
- $r_i(x, y)$  – The subsequent rasters for ordinary Euclidean distance.
- $s_i(x, y)$  – The subsequent rasters for weighted Euclidean distance.
- $t(x, y)$  – The temporary raster for cell-by-cell minimum of two distance rasters.
- $v(x, y)$  – The final raster for cell-by-cell minimum of all distance rasters.
- $w_i$  – The constant raster representing the weight value for feature  $i$ .

For  $i = 0$  to  $k - 1$

    If  $i = 0$  Then

- Calculate  $r_0(x, y)$
- $s_0(x, y) = r_0(x, y)/w_i$
- $v(x, y) = s_0(x, y)$

    Else

- Calculate  $r_i(x, y)$

$$s_l(x, y) = r_l(x, y)/w_l$$

$$t(x, y) = \text{Minimum}[v(x, y), s_l(x, y)]$$

$$v(x, y) = t(x, y)$$

EndIf  
Next *i*

It can be seen from the pseudo-code that only 7 rasters are required at any time, no matter how many features are involved in generating the weighted Voronoi diagram. The raster objects can be set to “Nothing” at the end of the process to free memory.

The result from Eq. (5) is a raster dataset representing WED to the sites. As shown in the 2-D profile in Fig. 4, the raster looks like a digital elevation model (DEM) in which basins are separated by ridges. Based on hydrological models, a flow direction raster can be calculated from the DEM, and then the basins can be obtained from the flow directions. The raster basins are then converted to vector polygons (Voronoi polygons) using raster to polygon conversion, and a spatial joining can be carried out to assign the non-spatial attributes of the input features to the Voronoi polygons. Fig. 5 shows the flowchart for generating ordinary or weighted Voronoi diagrams from point features (the same procedure can also be used for line and polygon features). This process is executed auto-

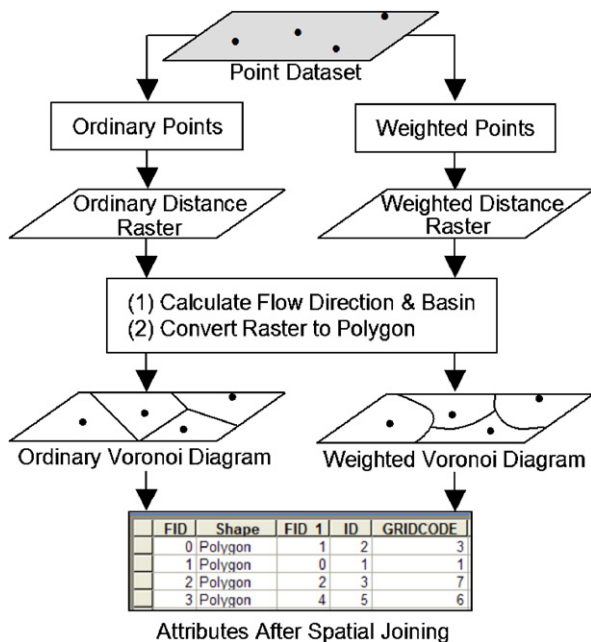


Fig. 5. Flowchart for generating ordinary or weighted Voronoi diagrams from point features.

matically and no user interaction is required (see Section 2.3).

## 2.2. Updating multiplicatively weighted Voronoi diagrams

Sometimes new features are collected and the existing Voronoi diagrams need to be updated. Since a distance raster needs to be calculated for each feature, it would be time-consuming if all features (both existing and new) are processed every time an update is needed, especially if there is a large number of existing features. The process of updating a Voronoi diagram is illustrated in Fig. 6 using point data as example (the process also works for line and polygon features). For example, a Voronoi diagram for 200 points has been generated from a distance raster (Distance Raster 1 in Fig. 6) for a study area, and the diagram needs to be updated using 3 new points (Dataset 2). Ideally, one would like to process only the 3 new points instead of 203 points. On one hand, 3 WED rasters can be created for the 3 new points using Eq. (4). The cell-by-cell minimum raster (Distance Raster 2) of the 3 WED rasters is then obtained using Eq. (5). Distance Raster 2 is compared with Distance Raster 1 to obtain the updated cell-by-cell minimum raster (Distance Raster 3). An updated Voronoi diagram is derived from Distance Raster 3. On the other hand, Dataset 2 is appended to Dataset 1 to obtain an updated Dataset 1, and the non-spatial attributes of the updated Dataset 1 can be assigned to the updated Voronoi diagram through spatial joining (Fig. 6).

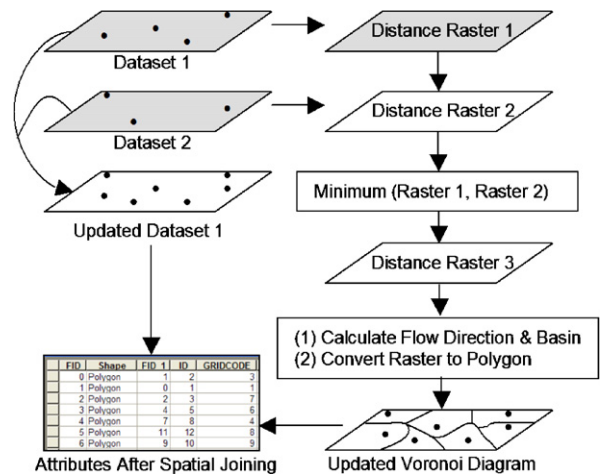


Fig. 6. Flowchart for updating Voronoi diagrams.



It should be noted that the above updating process can only update features within the extent (bounding-box or envelope) of existing features. Any new feature that is outside the existing extent will be ignored. If the extent of the new features is not completely contained by the existing extent, the program will generate a warning message. In that case, it is recommended to append the new features to existing features, and then generate new Voronoi diagrams as described in Section 2.1.

### 2.3. Implementation with ArcObjects

Written in C++, ESRI's ArcObjects are platform-independent software components for GIS application development. Several choices of customization are available for developers and users to add new GIS functionality to ArcGIS. Among the choices, extensions provide the developer with a

powerful mechanism for extending the core functionality of ArcGIS applications. An extension can provide new tools on a toolbar, listen for and respond to events, and perform many other GIS tasks. The weighted Voronoi diagram extension was developed using Microsoft Visual Basic 6.0 and ESRI's ArcObjects 9.1. From a Visual Basic ActiveX Dynamic Link Library (DLL) project, a DLL file was compiled. The DLL file can be deployed to a user's computer with other relevant files. Table 1 lists the components of the DLL project and major interfaces. The *IExtension* interface defines the name of the extension and specifies what action takes place when the extension is started or shut down. The *IExtensionConfig* interface specifies the state of the extension, and provides the name and description of the extension in the Extension dialog box.

The graphical user interface has 2 tabs: Generate (Fig. 7A) and Update (Fig. 7B). Using the Generate tab, the user can generate ordinary or weighted Voronoi diagrams from point, line, or polygon features. A raster showing OED or WED (if weights are used) is generated, as well as a Voronoi polygon shapefile. Non-spatial attributes from input features can be transferred to Voronoi polygons automatically through spatial joining. Once a distance raster is generated, the user can use new features to update the distance raster and create updated Voronoi polygons (Fig. 7B). The output Voronoi diagrams (shapefiles) and distance raster datasets can be used in spatial analysis and modeling for various

Table 1  
Components of visual basic ActiveX DLL project for weighted Voronoi diagram extension

Forms	frmVoronoi.frm
Modules	FunctionModules.bas
Class modules	Extension.cls (Implements: IExtension, IExtensionConfig) Help.cls (Implements: ICommand) StartVoronoi.cls (Implements: ICommand) Toolbar.cls (Implements: IToolbarDef)
Related documents	Voronoi.res (resource file)

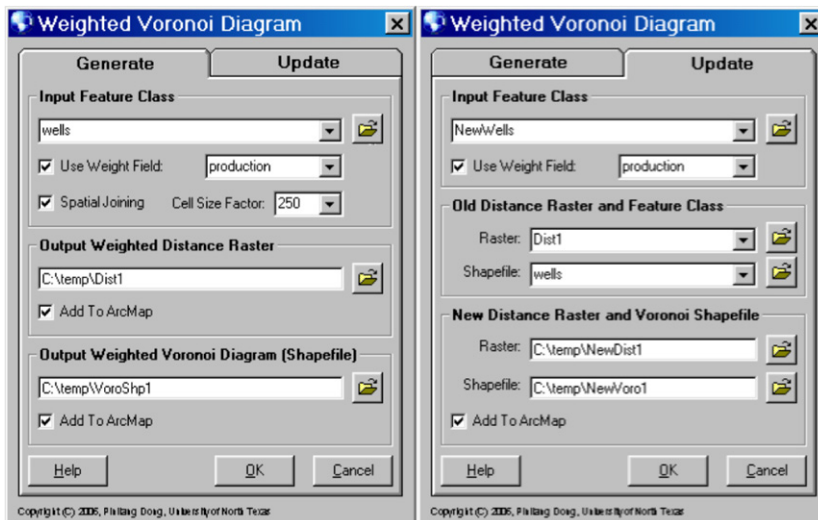


Fig. 7. Graphical user interface. (A) Generate tab; (B) Update tab.

applications. Examples of generating and updating ordinary and weighted Voronoi diagrams are provided in the next section. More examples and information on the functionality can be found in the user’s guide. The free extension and relevant documents are available from the author.

### 3. Examples

#### 3.1. Generating weighted Voronoi diagrams

In Fig. 8, the left side (Fig. 8A, C, E) shows the original data points and generated Voronoi diagrams. The numbers show the weight of the points. The right side of Fig. 8 shows WED rasters (Fig. 8B, D, F). In Fig. 8A, all the points have a weight 1, and the resulting Voronoi diagram is an OVD in which polygon sides are straight line segments. In fact, from Eq. (4) it is not difficult to prove that an OVD can be generated using a constant weight (not necessarily 1) for all features.

Fig. 8C and E show that weighted Voronoi diagrams will be created if the weights are different. Since these are multiplicatively weighted Voronoi diagrams, the weight ratio for 2 adjacent points is equal to the ratio of the distances from the points to the Voronoi boundary between them. For example, if we draw a line connecting the point with weight 0.8 with the point with weight 1.6 in Fig. 8E, we can see that the line is divided by the polygon boundary with a 1/2 ratio.

Fig. 9 shows a composite display of 10 line segments, their Voronoi diagrams, and the distance rasters. Fig. 9A is an OVD. In Fig. 9B, the length field of the line attribute table was used as weight field to create a weighted Voronoi diagram. Compared with Fig. 8A, it can be seen that shorter line segments in Fig. 9B have smaller Voronoi polygons.

Similar to Fig. 9, Fig. 10 shows a composite display of 10 polygons, their Voronoi diagrams, and the distance rasters. Fig. 10A is an OVD, and

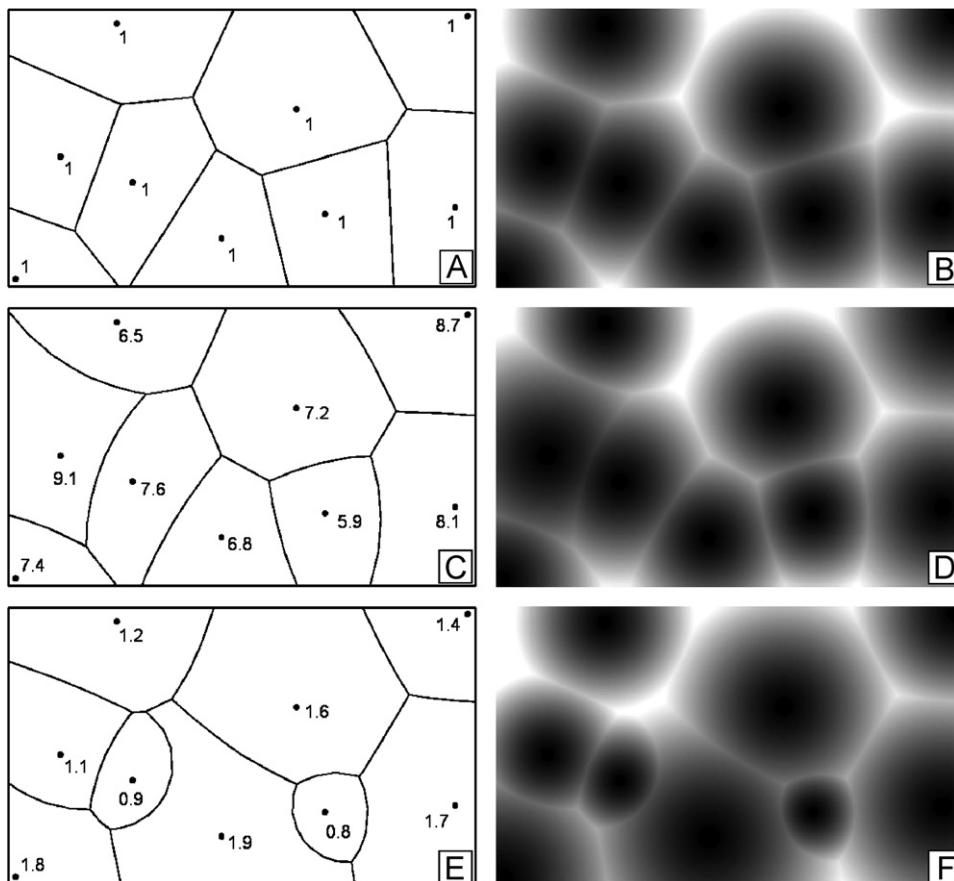


Fig. 8. Examples of weighted Voronoi diagrams and distance rasters for point features.

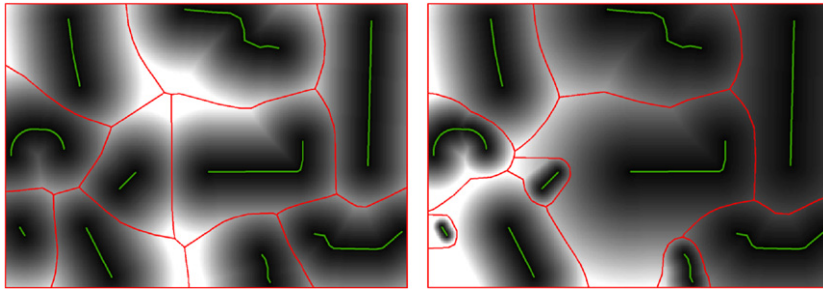


Fig. 9. Voronoi diagrams for lines. (A) Ordinary Voronoi diagram; (B) weighted Voronoi diagram using line length as weight.

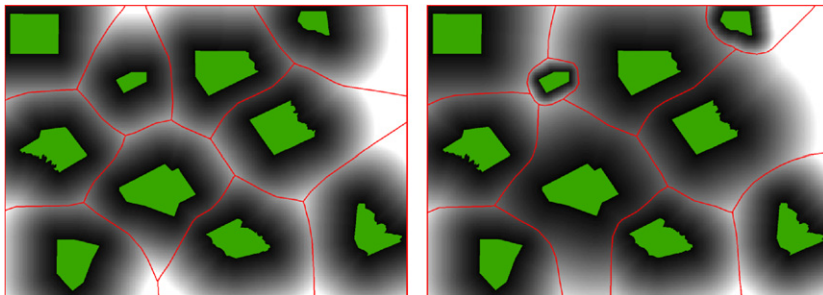


Fig. 10. Voronoi diagrams for polygons. (A) Ordinary Voronoi diagram; (B) weighted Voronoi diagram using polygon area as weight.

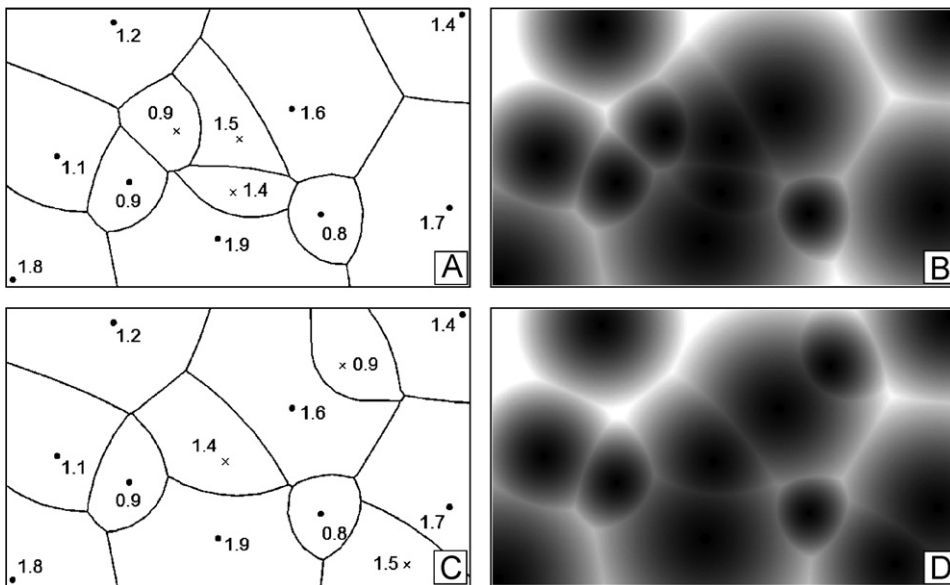


Fig. 11. Updating Voronoi diagrams. (A) and (C) show original points (dot symbols), new points (cross symbols), and updated Voronoi diagrams; (B) and (D) are weighted distance rasters.

Fig. 9B is a weighted Voronoi diagram using feature area as weight. Compared with Fig. 9A, it can be seen that smaller polygons in Fig. 9B have smaller Voronoi polygons.

### 3.2. Updating weighted Voronoi diagrams

The following examples show how the weighted Voronoi diagram in Fig. 8E can be updated by 3



additional points. This process requires the original points (dots in Fig. 8E) and the original weighted distance raster (Fig. 8F). Three new points with weights 0.9, 1.4, and 1.5 (x symbols in Fig. 11A) are appended to the original points to create a new Voronoi diagram (polygons in Fig. 11A). The updated weighted distance raster is shown in Fig. 11B. Fig. 11C and D show the updated weighted Voronoi diagram and weighted distance raster when the 3 new points (x symbols in Fig. 11C) are added to 3 different locations.

#### 4. Discussions

Some advantages and limitations of the extension should be discussed. The advantages are as follows:

- (1) it works seamlessly with ArcGIS, and it can handle point, line, and polygon features;
- (2) it can generate both ordinary and multiplicatively weighted Voronoi diagrams. The weights for multiplicatively weighted Voronoi diagrams can be from an existing numeric field in the attribute table, or from a new numeric field with user-specified values;
- (3) the input non-spatial attributes can be automatically assigned to output Voronoi polygons through spatial joining;
- (4) Voronoi diagrams generated by this extension can be conveniently updated by inserting additional generators (points, lines, or polygons); and
- (5) in addition to ordinary or weighted Voronoi diagrams in vector format, this extension also creates ordinary or WED rasters. While the Voronoi diagrams partition the space into

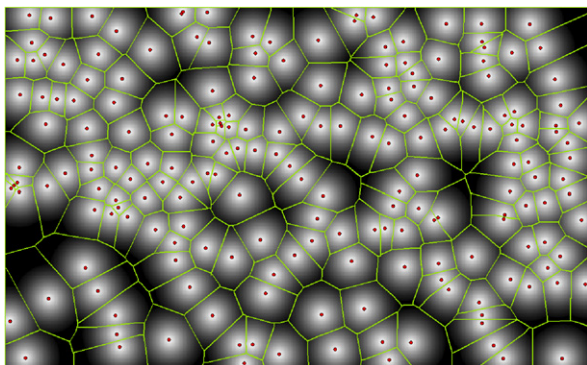


Fig. 12. An ordinary Voronoi diagram for 200 points (with negative distance raster as background).

Voronoi cells, the distance rasters show the distance variations within each Voronoi cell, which can be very useful for spatial modeling. Fig. 12 displays 200 points, their OVD, and the negative distance raster (with brighter tones representing locations closer to a generator point).

Some existing or possible issues with the extension are also addressed here:

- (1) If a large number of input features are involved, the process for generating weighted Voronoi diagrams can be relatively slow because that process loops through all features one by one. The author created a multiplicatively weighted Voronoi diagram for the 200 points in Fig. 12 in about 15 min (Windows 2000 on Intel Pentium 4, 3.2 GHz, 2G RAM). For many GIS applications, this speed is acceptable. For generating OVDs, there is no need to loop through all features for distance calculation, and the processing is much faster (it takes only several seconds to process the same 200 points).
- (2) Since the major processing steps are raster-based, it is possible that an input feature does not have an output Voronoi polygon if that feature is very close to other features and raster cell size is not small enough. This problem can be solved by specifying a larger “cell size factor” on the user interface to reduce the cell size. The cell size is the shortest of the width or height of the extent of input feature layer, in the output spatial reference, divided by the cell size factor (default to 250). The geoprocessing environmental settings are updated in a code based on user selections.
- (3) Some Voronoi programs outside GIS can handle point insertion and deletion to update Voronoi diagrams. This extension can handle insertion of new features to update a Voronoi diagram. However, since distance rasters for individual features are created temporarily (not saved to the hard disk permanently), feature deletion is not supported when updating Voronoi diagrams because old temporary rasters cannot be retrieved. As many GIS applications do not need real-time generation of Voronoi diagrams, this should not be a major limitation. Indeed, features can be easily edited in GIS before Voronoi diagrams are generated.

It should be noted that the references in this paper and the topic of this paper are all on planar Voronoi diagrams generated from Euclidean distance. As a side note, it is fascinating to see that OVDs have been generated for networks using the shortest-path distance, as demonstrated in the software SANET developed by Okabe et al.<sup>4</sup> Future versions of SANET will be able to create weighted Voronoi diagrams for networks (Okabe, 2007, personal communication).

## 5. Conclusion

A raster-based approach to generating and updating ordinary and multiplicatively weighted Voronoi diagrams is developed and implemented as an extension in ArcGIS. It works seamlessly with ArcGIS, and can handle point, line, and polygon features. It can perform spatial joining to link input features and output Voronoi polygons. In addition to Voronoi diagrams in vector format, the extension also creates ordinary or WED rasters. The Voronoi diagrams and distance rasters can be conveniently combined with other GIS datasets to support both vector-based spatial analysis and raster-based spatial modeling. The analyses of the advantages and limitations of the extension suggest that it can meet the requirements of many GIS applications, and that it can significantly improve the integration of Voronoi diagram models and GIS applications.

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